

# Advertising, in-house R&D, and growth

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This paper develops a quality-ladder model of endogenous growth to study the interplay between in-house R&D and combative advertising expenditure, and its implications for economic growth, firm size, and welfare. The analysis shows that, somewhat surprisingly, higher incentives to engage in advertising, although combative, unambiguously foster innovation activity of firms. This, possibly, leads to faster growth and even higher welfare. These results rest on two features of the model which are well-supported by empirical evidence. First, if firms incur higher sunk costs for marketing, concentration and firm size rise. Second, firm size and R&D expenditure are positively related as larger firms are able to spread R&D costs over higher sales. The analysis also suggests that R&D subsidies are conducive to R&D and growth without inducing firms to raise advertising outlays.

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## 1. Introduction

Firms devote large amounts of resources to advertising. For instance, for consumer products, the US advertising-to-sales ratio in 2002 was 6.7% on average, and still 2.9% for all sectors combined (Schonfeld & Associates, 2003). For the UK, the mean advertising-to-sales ratio in 1999 was 3.3% for consumer manufacturing and 2.4% for all industries (Paton and Conant, 2001).

Different views on the role of advertising for social welfare have emerged in the literature. For instance, it has been argued that advertising may play a constructive role of providing information about existence and characteristics of products to consumers (e.g., Nelson, 1974; Grossman and Shapiro, 1984) or may be valued directly (being perceived as product characteristic itself) even when uninformative (Stigler and Becker, 1977; Becker and Murphy, 1993). The more traditional view emphasizes that advertising can be socially wasteful by merely redistributing demand from low-advertisers to firms which engage in advertising more heavily (e.g., Pigou, 1929; Kaldor, 1950). That is, advertising is combative in the sense that an increase in marketing expenditure of a single firm creates a negative externality on the demand faced by other firms.

As pointed out by Bagwell (2005) in a comprehensive review of the literature on the economic effects of advertising, 'no single view of advertising is valid in all circumstances' (p.54). Some empirical studies provide support for the informative view on advertising, suggesting that advertising directly enhances welfare. However, it is also fair to conclude that 'advertising is often combative in nature' (Bagwell, 2005, p.30). This notion gets considerable support not only in earlier studies (e.g. Comanor and Wilson, 1967, 1974; Lambin, 1976), but also in the more recent literature on advertising which pursues a behavioral economics approach. The latter strand emphasizes strategies of firms to distract attention from other firms' products, to frame product characteristics in a favorable way (like shrouding negative attributes), elicit emotions of consumers, and other ways to distort tastes (e.g., Levin and Gaeth, 1988; Bertrand *et al.*, 2004; Gabaix and Laibson, 2006).

According to Kaldor (1950), if advertising is combative, normative justification must come from indirect effects it causes. This paper argues that even combative advertising may have positive net effects on welfare because it positively interacts with R&D spending of firms. For this purpose, a quality-ladder model of endogenous growth with free entry of firms is developed in which both in-house R&D and marketing expenditure are essential elements for firms to successfully compete in monopolistic product markets. Combative advertising is reflected by the standard feature that for a given number of firms, the market share of a single firm depends on the amount of advertising expenditure it incurs relative to its rivals (see e.g., Schmalensee, 1972; Bell *et al.*, 1975; Barros and Sørsgard, 2001; Baye and Morgan, 2005).

The basic mechanism which gives rise to potentially positive growth and welfare effects of advertising is the following: higher marketing outlays per firm in the economy, because they add to sunk costs and therefore foster concentration, raise market shares of firms for any given distribution of R&D expenditure. In turn, since the cost of any unit of R&D investment is spread over higher sales ('cost spreading'), the return to R&D and thus R&D effort per firm rises.

This mechanism is supported by empirical evidence. First, the evidence lends support to the Kaldorian view of a positive relationship between advertising expenditure levels (as shares of total sales revenues) and industry concentration (e.g., Mueller and Rogers, 1984; Sutton, 1991). In the words of Kaldor (1950, p.13), 'after advertising has been generally adopted . . . sales will have been concentrated among a smaller number of firms, and the size of 'representative firm' will have increased'. Consistent with this notion, Sutton (1998, p.221) concludes that in the pharmaceutical industry, 'marketing efforts . . . raise the fixed outlays occurred in bringing a drug to market, and thus raises the . . . level of concentration'. Second, as hypothesized by Schumpeter (1942) and formalized in the endogenous growth literature featuring in-house R&D (Smulders and van de Klundert, 1995; Peretto, 1998, 1999; Young, 1998), firm size and R&D expenditure are strongly positively related (for evidence, see e.g. Cohen and Levin, 1989, Cohen and Klepper, 1996a,b,

and the references therein). As stated in Cohen and Klepper (1996a, p.929) ‘... in most industries, it has not been possible to reject the null hypothesis that R&D varies proportionally with size across the entire firm size distribution’.<sup>1</sup> Cohen and Klepper (1996a) also show that a positive size-R&D relationship arises at the business unit level, which excludes alternative explanations relying on imperfect capital markets or scope economies. Importantly, their evidence supports the implication of the cost-spreading explanation that the positive size-R&D link should be weaker if inventions are more saleable. Moreover, Pagano and Schivardi (2003) find a positive and robust relationship between average firm size and productivity growth within industries by analyzing a data set of eight European countries. They show that this relationship is stronger in R&D intensive industries (in terms of an external sectorial ranking, i.e., R&D intensity in the US). Therefore, they conclude that a positive impact of firm size on productivity growth is not due to reverse causality.

The paper is organized as follows. Section 2 sets up the basic structure of the model. Section 3 analyses the equilibrium, in particular, examining whether advertising competition among firms crowds out innovation activity of firms or complements it. Section 4 extends the model in two directions. It first shows that the magnitude of scale effects with respect to growth plays a critical role for growth and welfare effects of advertising. Secondly, it explores the implications of R&D subsidies for the key variables. Section 5 checks robustness of the main results. The framework of analysed in Sections 2–4 is one of differentiated consumer goods, without physical capital, and with a demand structure which is characterized by a constant price elasticity (as common in the endogenous growth literature). In contrast, the first part of Section 5 introduces advertising and R&D into the alternative linear-demand monopolistic competition framework of Ottaviano *et al.* (2002). The second part proposes a model where R&D interacts with capital accumulation and there is advertising on differentiated producer goods. Both alternative models lead to similar results as the basic model. Section 6 concludes. All proofs are relegated to the Appendix.

## 2. The basic model

Consider an economy which is populated by  $L$  individuals with infinite lifetimes, each supplying one unit of labor in each period  $t = 0, 1, 2, \dots$  (i.e., there is no population growth). The labor market is perfect and the wage rate is normalized to unity,  $w_t = 1$ . There is a representative consumer with intertemporal utility function

$$U = \sum_{t=0}^{\infty} \rho^t \ln C_t, \quad (1)$$

<sup>1</sup> According to OECD (1999, Table 5.4.1), in 1997, 84.7% of business R&D expenditure in the US and about two thirds in most other countries have been incurred by firms with more than 500 employees. Notably, the fact that R&D is concentrated in large firms is not due to a government bias of public R&D financing towards larger firms. The share of government-financed business R&D is rarely over 10% and in some countries even biased to small companies (e.g., in Belgium, Finland, and Switzerland).

$0 < \rho < 1$ .  $C_t$  is a consumption index, which is given by

$$C_t = \left( \int_0^{n_t} (q_t(i)x_t(i))^{\sigma-1/\sigma} di \right)^{\sigma/\sigma-1}, \tag{2}$$

$\sigma > 1$ , where  $x_t(i)$  denotes the quantity of variety  $i \in \mathcal{N}_t = [0, n_t]$  consumed in period  $t$ , and  $q_t(i)$  indicates its quality.

Each firm produces one variety of the horizontally differentiated product in monopolistic competition. The production function is given by

$$x_t(i) = l_t^P(i), \tag{3}$$

where  $l_t^P(i)$  denotes the amount of production-related labor employed in firm  $i \in \mathcal{N}_t$  at date  $t$ . The measure  $n_t$  is referred to as the number of firms in  $t$  and is endogenously determined for  $t \geq 1$ , whereas  $n_0$  is historically given.

Following Young (1998), firms can incur in-house R&D labor investments in order to improve product quality one period in advance of production.<sup>2</sup> Moreover, and in contrast to previous models of endogenous technical change, firms may promote their products and innovations by incurring advertising expenditure. More specifically, suppose that success in the product market depends on a firm’s advertising expenditure relative to that of its rivals. Thus, marketing activity creates negative externalities on product demand among firms. Formally, we distinguish between true quality,  $q_t(i)$ , and perceived quality,  $q_t^*(i)$ , where the latter is relevant for consumers’ choice. Let  $l_{t-1}^R(i)$  and  $l_{t-1}^M(i)$  denote the amount of R&D and marketing labor employed by firm  $i \in \mathcal{N}_t$  in  $t-1$ , respectively. Denote by  $\bar{l}_{t-1}^M = (1/n_t) \int_0^{n_t} l_{t-1}^M(i) di$  the average amount of marketing labor and by  $m_{t-1}(i) = l_{t-1}^M(i)/\bar{l}_{t-1}^M$  the relative marketing expenditure of firm  $i$ ,  $t \geq 1$ . Perceived product quality  $q_t^*(i)$  of variety  $i$  in any period  $t \geq 1$  evolves according to

$$q_t^*(i) = q_t(i)m_{t-1}(i)^\eta, \text{ where } q_t(i) = \bar{S}_{t-1} l_{t-1}^R(i)^\kappa. \tag{4}$$

$\kappa > 0$  and  $\eta \geq 0$  measure the effectiveness of R&D and marketing, respectively, and  $\bar{S}_{t-1}$  reflects the state of technology in  $t-1$ . This technology implies that in symmetric equilibrium, where  $m(i) = 1$  for all  $i$ , perceived and true quality coincide. As demonstrated in a supplement to this paper (available on request), the complementarity between true quality and marketing input implied by

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<sup>2</sup>The benchmark model without marketing roughly follows Young (1998) in the sense that long-run growth is fueled by in-house R&D with positive knowledge spillovers. There is neither a ‘creative destruction/business stealing’ effect (e.g., Aghion and Howitt, 1992) nor a ‘stepping on toes’ effect (Jones and Williams, 2000) in the model, which would distort R&D decisions towards overinvestment. Calibration exercises by Jones and Williams (2000) and Alvarez and Groth (2005) indeed suggest that knowledge spillovers dominate the other externalities from R&D.

specification (4) is inconsequential for the main results and chosen merely for analytical convenience.

Several remarks are in order. First, our formulation captures that firms engage in a combative advertising contest. Skaperdas (1996) provides an axiomatic foundation of contest success functions, where relative resources incurred affect the probability of success. Here, for simplicity, a deterministic relationship between a firm's relative advertising spending and its demand faced is supposed. As shown in the working paper version of this article (Grossmann, 2003), uncertainty of the success of marketing activity can be introduced without affecting the main results. Relying on contest success functions in the context of advertising has a long tradition in both economics (see e.g., Schmalensee, 1972, ch. 2) and in the marketing literature (e.g., Bell *et al.*, 1975).<sup>3</sup> It reflects the Kaldorian view that advertising, since pursued by an interested party, largely tries to persuade rather than to inform consumers and therefore is combative in nature. The notion that advertising is often combative has been supported by a large body of empirical evidence. For instance, Lambin (1976) shows that sales and market share of a firm, although increasing with own advertising outlays, drops considerably when rivals raise their advertising expenditure. As a result, the impact of advertising on total industry sales is limited, consistent with the notion of combative advertising. Recent evidence provided by behavioral economists has shed light on the way how consumers' tastes are distorted by advertising (Bertrand *et al.*, 2004; Gabaix and Laibson, 2004). This is not to deny, however, that advertising is often informative and therefore directly welfare-enhancing. Our focus on combative advertising primarily serves to highlight that growth and welfare may be positively related to advertising expenditures of firms, even in an environment in which advertising has no direct welfare-enhancing effects. Also, examining implications of non-informative advertising allows us to abstract from informational asymmetries which keeps the analysis reasonably simple.

Long-run growth is driven by an intertemporal spillover through which knowledge accumulates. Specifically, suppose that the state of technology in  $t-1$  is given by the average product quality of firms active in  $t-1$ ,  $\bar{q}_{t-1}$ , i.e.,

$$\bar{S}_{t-1} = \bar{q}_{t-1} = \frac{1}{n_{t-1}} \int_0^{n_{t-1}} q_{t-1}(i) di. \quad (5)$$

That is, knowledge acquired by R&D activity is private information of a firm for one period (e.g., due to intellectual property rights). For concreteness, assume that in the initial period product quality of any firm is given by  $q_0(i) = \bar{S}_0 > 0$ ,  $i \in \mathcal{N}_0$ . According to (5), if all firms invest the same amount of labor at date  $t-2$  in R&D ( $I_{t-2}^R(i) = I_{t-2}^R$ ) and therefore  $q_{t-1}(i) = \bar{S}_{t-1} (I_{t-2}^R)^k$  for all  $i$ , the number of firms

<sup>3</sup> More recently, Barros and Sørgard (2001) examine the implications of mergers in the presence of an advertising contest. Baye and Morgan (2005) capture creation of brand loyalty in online markets by a contest among firms.

which invest in period  $t - 2$  and produce final output in  $t - 1$ ,  $n_{t-1}$ , does not matter for research capabilities of firms in the subsequent period. This assumption reflects the notion of Young (1998) that innovations of firms are ‘equivalent’ in the sense that firms come up with similar solutions to similar problems at the same time. As will become apparent, this eliminates the often undesired feature of many endogenous growth models that the economy’s growth rate depends on population size  $L$  (‘scale effect’).<sup>4</sup> Section 4.1 further discusses this issue and provides an extension which allows for scale effects.

In order to capture that in the long-run the number of firms should adjust to changes in sunk cost expenditure levels, there is free entry of firms into the economy, with a large number of potential entrants. As will become apparent, endogeneity of the number of firms, and therefore of market concentration, is critical for the main results. Suppose that at all times, firms have to incur a fixed labor requirement  $f > 0$  prior to production, which may be thought of being related to red tape or the organization of production. Note that, as  $f$  has to be incurred each period and the intertemporal spillover effect cannot be appropriated by firms, each firm’s planning horizon is exactly one period in advance (Young, 1998). In  $t - 1$ , each firm  $i \in \mathcal{N}_t$  (producing final output in period  $t$ ) issues bonds or shares in a perfect financial market in order to finance fixed costs  $f$ , as well as R&D and marketing investments,  $l_{t-1}^R(i)$  and  $l_{t-1}^M(i)$ , respectively.

### 3. Equilibrium analysis

According to (2), for a given aggregate consumption expenditure  $E_t$ , the demand function for good  $i$  in period  $t$  under perceived quality  $q_t^*(i)$  is given by

$$x_t^D(i) = q_t^*(i)^{\sigma-1} \frac{E_t}{P_t} \left( \frac{p_t(i)}{P_t} \right)^{-\sigma}, \tag{6}$$

where  $p_t(i)$  is the price of good  $i$  in  $t$  and  $P_t$  is a price index, defined as

$$P_t \equiv \left( \int_0^{n_t} \left( \frac{p_t(i)}{q_t^*(i)} \right)^{1-\sigma} di \right)^{1/1-\sigma}. \tag{7}$$

In the case where  $q_t^*(i) = q_t(i)$  for all  $i$  (which will hold in equilibrium), this implies  $C_t = E_t/P_t$ , i.e.,  $C_t$  equals ‘real’ consumption expenditure in period  $t$ . For given initial asset holding,  $A_0 > 0$ , asset holding of the representative consumer evolves according to

$$A_{t+1} = (1 + r_t)A_t + L - E_t, \tag{8}$$

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<sup>4</sup> See also Jones (1995a), Dinopoulos and Thompson (1998), and Segerstrom (1998) for endogenous growth models without scale effects regarding the growth rate, each highlighting different aspects.

where  $r_t$  denotes the interest rate in  $t$  and  $r_0$  is given. (Note that  $L$  equals aggregate wage income, as  $w_t = 1$  and labor supply is fixed at  $L$ .) Since  $C_t = E_t/P_t$ , under logarithmic utility (1), the intertemporal utility optimization problem implies that, for all  $t \geq 1$ , consumption spending evolves according to the Euler equation

$$E_t = (1 + r_t)\rho E_{t-1}. \tag{9}$$

Profits of firm  $i$  in period  $t$  are given by  $\pi_t(i) = (p_t(i) - 1)x_t^D(i)$ . Then, at time  $t - 1$ , each firm  $i \in \mathcal{N}_t$  chooses non-production labor investments  $l_{t-1}^R(i)$  and  $l_{t-1}^M(i)$  to maximize its firm value  $\pi_t(i)/(1 + r_t) - l_{t-1}^R(i) - l_{t-1}^M(i) - f$ , which can be written as

$$\frac{p_t(i) - 1}{1 + r_t} \left[ \bar{S}_{t-1} l_{t-1}^R(i)^{\kappa} \left( \frac{l_{t-1}^M(i)}{\bar{l}_{t-1}^M} \right)^{\eta} \right]^{\sigma-1} \frac{E_t}{P_t} \left( \frac{p_t(i)}{P_t} \right)^{-\sigma} - l_{t-1}^R(i) - l_{t-1}^M(i) - f, \tag{10}$$

according to (4) and (6). Note that  $P_t$ ,  $E_t$  and  $\bar{l}_{t-1}^M$  are taken as given in the optimization problem of firms.

*Definition 1* (Equilibrium) An equilibrium is a sequence  $\{E_t, n_{t+1}\}$  of aggregate consumption spending and the number of firms, a sequence  $\{r_{t+1}, P_t\}$  of interest rates and price indices, a sequence  $\{x_t^D(i), l_t^P(i), p_t(i)\}$  of product demand, production employment levels and output prices of firms  $i \in \mathcal{N}_t$ , and a sequence of R&D employment, absolute and relative marketing employment as well as perceived and actual product quality  $\{l_t^R(i), l_t^M(i), m_t(i), q_{t+1}^*(i), q_{t+1}(i)\}$  of firms  $i \in \mathcal{N}_{t+1}$ ,  $t = 0, 1, 2, \dots$ , which satisfy the following conditions:

- (E1) Given the sequences  $\{r_t\}$ ,  $\{n_t\}$ ,  $\{p_t(i)\}$  and  $\{q_t^*(i)\}$ ,  $i \in \mathcal{N}_t$ , the sequences  $\{E_t\}$  and  $\{x_t^D(i)\}$ ,  $i \in \mathcal{N}_t$ , maximize the representative household's utility (1), (2) subject to  $E_t = \int_0^{n_t} p_t(i)x_t(i)di$  and (8).
- (E2) Given  $p_t(i)$ ,  $P_t$ ,  $\bar{l}_{t-1}^M$ ,  $E_t$ ,  $r_t$  and  $\bar{S}_{t-1}$ , for any  $t \geq 1$ ,  $l_{t-1}^R(i)$  and  $l_{t-1}^M(i)$  maximize firm value (10) and given  $P_t$ ,  $E_t$  and  $q_t^*(i)$ , for any  $t \geq 0$ ,  $p_t(i)$  maximizes  $(p_t(i) - 1)x_t^D(i)$  s.t. (6),  $i \in \mathcal{N}_t$ .
- (E3) For any  $t \geq 1$ , the firm value (10) at  $(p_t(i), l_{t-1}^R(i), l_{t-1}^M(i))$ ,  $i \in \mathcal{N}_t$ , equals zero (free entry).
- (E4) For any  $t \geq 0$ ,  $x_t^D(i) = l_t^P(i)$ ,  $i \in \mathcal{N}_t$  (goods market equilibrium).
- (E5) For any  $t \geq 0$ ,  $\int_0^{n_t} l_t^P(i)di + \int_0^{n_{t+1}} (l_t^R(i) + l_t^M(i))di + n_{t+1}f = L$  (labor market equilibrium).
- (E6) For any  $t \geq 0$ ,  $q_{t+1}(i)$  and  $q_{t+1}^*(i)$  are given by (4),  $m_t(i) = l_t^M(i)/\bar{l}_t^M$ ,  $i \in \mathcal{N}_{t+1}$ , and  $P_t$  is given by (7); finally, for any  $t \geq 2$ ,  $\bar{S}_{t-1}$  is given by (5).<sup>5</sup>

<sup>5</sup>According to Walras' law, the equilibrium conditions in Definition 1 imply that also the asset market clears. The sequences  $\{C_t\}$  and  $\{x_t(i)\}$ ,  $i \in \mathcal{N}_t$ , immediately follow from (2) and (3), respectively.

The household's problem (E1) is solved by (6)-(9). Regarding the firms' problem (E2), we first consider price setting. The isoelastic demand functions (6) lead to constant mark-ups over marginal production costs (which are equal to unity); we have  $p_t(i) = \sigma/(\sigma - 1) \equiv \tilde{p}$  for all  $t \geq 0$ . Since firms are identical *ex ante*, the analysis focusses on an equilibrium with symmetric non-production employment levels for R&D and marketing activities, i.e.,  $l_{t-1}^R(i) = \bar{l}_{t-1}^R$  and  $l_{t-1}^M(i) = \bar{l}_{t-1}^M (= \bar{l}_{t-1}^M)$  for all  $i \in \mathcal{N}_t$ ,  $t \geq 1$ . Thus,  $x_t^D(i) = x_t^D$  for all  $i$ . It is straightforward to show (available on request) that objective function (10) is strictly concave as a function of  $(l_{t-1}^R(i), l_{t-1}^M(i))$  if and only if

$$(\kappa + \eta)(\sigma - 1) < 1, \tag{11}$$

which is assumed hereafter. Using (10), under symmetry, the first-order conditions regarding optimal R&D and marketing effort are given by

$$\frac{\tilde{p} - 1}{1 + r_t} x_t^D (\sigma - 1) \frac{\kappa}{\bar{l}_{t-1}^R} \leq 1, \tag{12}$$

$$\frac{\tilde{p} - 1}{1 + r_t} x_t^D (\sigma - 1) \frac{\eta}{\bar{l}_{t-1}^M} \leq 1, \tag{13}$$

with equality if  $l_{t-1}^R > 0$  and  $l_{t-1}^M > 0$ , respectively. The left-hand sides of (12) and (13) equal the marginal benefit of R&D and marketing employment, respectively, whereas the right-hand sides equal marginal costs (recall  $w_t = 1$ ). If  $l_{t-1}^R > 0$  and  $l_{t-1}^M > 0$ , first-order conditions (12) and (13) imply

$$\frac{\bar{l}_{t-1}^M}{\bar{l}_{t-1}^R} = \frac{\eta}{\kappa}. \tag{14}$$

Hence, the ratio of marketing employment to R&D employment in any firm is time-invariant. It is given by the ratio of the elasticities of product demand,  $x_t^D(i)$ , with respect to R&D and marketing investments (the latter being evaluated at  $l_{t-1}^M(i) = \bar{l}_{t-1}^M$ ), which read  $\kappa(\sigma - 1)$  and  $\eta(\sigma - 1)$ , respectively. In a similar fashion as in Young (1998), the following can be shown. (All results are proven in the Appendix.)

*Lemma 1* The equilibrium interest rate immediately jumps to a steady state level, with  $r_t = (1 - \rho)/\rho \equiv \tilde{r}$  for all  $t \geq 1$ .

The absence of transitional dynamics in the model is due to the linear spillover effect in the evolution of perceived quality (4). This feature of the model greatly simplifies the analysis of welfare effects.

It turns out that existence of a symmetric equilibrium, which shall be our exclusive focus, requires parameter configurations such that goods quality levels improve. As is apparent from the next result, this is ensured if

$$f > \frac{1 - (\kappa + \eta)(\sigma - 1)}{\kappa(\sigma - 1)} \equiv \hat{f}. \tag{15}$$



*Proposition 1* (Symmetric equilibrium). In symmetric equilibrium, the following holds.

(i) For any  $t \geq 1$ , R&D labor and marketing labor per firm are given by

$$l_{t-1}^R = \frac{\kappa(\sigma - 1)f}{1 - (\kappa + \eta)(\sigma - 1)} \equiv \tilde{l}^R \quad (16)$$

and

$$l_{t-1}^M = \frac{\eta(\sigma - 1)f}{1 - (\kappa + \eta)(\sigma - 1)} \equiv \tilde{l}^M, \quad (17)$$

respectively, and the number of firms is

$$n_t = \frac{\rho L [1 - (\kappa + \eta)(\sigma - 1)]}{f(\sigma - 1 + \rho)} \equiv \tilde{n}. \quad (18)$$

(ii) For any  $t \geq 2$ , the (approximate) growth rate  $\vartheta_t = \ln(c_t/c_{t-1})$  of real consumption per capita,  $c_t = C_t/L$ , is given by

$$\vartheta_t = \kappa \ln \tilde{l}^R = \kappa \ln \left( \frac{\kappa(\sigma - 1)f}{1 - (\kappa + \eta)(\sigma - 1)} \right) \equiv \tilde{\vartheta}. \quad (19)$$

(iii) Intertemporal utility ( $\tilde{U}$ ) is given by

$$\tilde{U} = \frac{\rho}{1 - \rho} \left( \frac{1}{\sigma - 1} \ln \tilde{n} + \frac{1}{1 - \rho} \tilde{\vartheta} \right) + \Lambda, \quad (20)$$

where  $\Lambda \equiv (\sigma - 1)^{-1} \ln n_0 + (1 - \rho)^{-1} \ln [\tilde{S}_0(\sigma - 1)L/(\sigma - 1 + \rho)]$ .

According to (20), welfare can be subdivided in two main components. First,  $\tilde{U}$  directly depends on the equilibrium number of firms  $\tilde{n}$ , given by (18), due to the ‘love-of-variety’ property of preferences (Dixit and Stiglitz, 1977). Second,  $\tilde{U}$  is positively related to the (approximate) growth rate  $\tilde{\vartheta}$  of the economy, which is given by (19). In fact, not only real consumption but also product quality grows at rate  $\vartheta_t = \tilde{\vartheta}$  from period 2 onwards. Note that  $\tilde{\vartheta}$  is independent of  $L$ , i.e., growth does not exhibit scale effects.

We now derive comparative-static results for changes in the effectiveness of marketing and R&D,  $\eta$  and  $\kappa$ , respectively. Changes in  $\eta$  are of particular interest. Since  $\tilde{l}^M = 0$  if and only if  $\eta = 0$ , according to (17),  $\eta = 0$  serves as a benchmark case. Thus, by considering changes in  $\eta$  one can examine, for instance, whether marketing possibilities crowd out R&D investments, in turn reducing growth, or if they complement innovation activity. In addition, changes in  $\kappa$  are considered to obtain further insights regarding the interplay between R&D and marketing incentives.

*Proposition 2* (Comparative-static results)

(i) An increase in the effectiveness of marketing or R&D,  $\eta$  or  $\kappa$ , respectively, raises both R&D and marketing labor per firm,  $\tilde{l}^R$  and  $\tilde{l}^M$ , as well as growth rate  $\tilde{\vartheta}$ , but reduces the number of firms,  $\tilde{n}$ .

(ii) An increase in  $\eta$  raises (does not affect, lowers) welfare,  $\tilde{U}$ , if  $\kappa(\sigma - 1) + \rho > (=, <) 1$ . Moreover, if  $\eta(\sigma - 1) \leq \rho$  and  $f > \hat{f}$ ,  $\tilde{U}$  is increasing in  $\kappa$ ; if  $\eta(\sigma - 1) > \rho$ , the impact of an increase in  $\kappa$  on  $\tilde{U}$  is ambiguous.

To understand part (i) of Proposition 2, note that an increase in  $\eta$  or  $\kappa$  raises the incentive of firms to incur sunk cost for marketing and R&D, respectively, for any given number of firms,  $n$ . This has a negative impact on the firm value, implying that less firms enter the economy, i.e.,  $\tilde{n}$  declines. Moreover, an increase in  $\kappa$  raises the amount of researchers per firm,  $\tilde{l}^R$ , and in view of (19), also raises the growth rate of real consumption,  $\tilde{\vartheta}$ . Moreover, advertising expenditure per firm,  $\tilde{l}^M$ , rises with  $\eta$ . These results are straightforward. What is more surprising at the first glance is that an increase in  $\eta$  unambiguously raises R&D employment per firm,  $\tilde{l}^R$ . (Similarly, an increase in  $\kappa$  raises  $\tilde{l}^M$ .) This result is driven by the following mechanism, which is well-supported by empirical evidence (as outlined in the introduction). First, due to free entry and the sunk cost nature of advertising outlays, a simultaneous increase in advertising outlays of all firms triggers a rise in product demand per firm,  $x^D$ . In turn, the return to R&D (given by the left-hand side of (12)) increases, as R&D costs can be spread over a larger amount of output. This induces firms to increase R&D investments, i.e., firm size and R&D are positively related.<sup>6</sup> Consequently, under specification (5) of the knowledge spillover, which removes scale effects with respect to growth, not only  $\tilde{l}^R$  but also the economy's growth rate  $\tilde{\vartheta}$  is rising in  $\eta$ , according to (19).

Three remarks are in order. First, it is important to stress that the endogeneity of the number of firms is critical for the result that R&D investment per firm rises as  $\eta$  (and thus advertising expenditure) increases. To see this, suppose to the contrary that the number of firms,  $n$ , is exogenously given. It is straightforward to show that, in this case, equilibrium R&D investment is decreasing rather than increasing in  $\eta$  (see Remark 1 in the Appendix for a formal derivation). This is because, when  $n$  is fixed, the only effect of advertising is a wasteful reallocation of labor resources, whereas market shares and therefore R&D incentives are unaffected.

A second remark concerns the property that the growth rate is basically determined by average R&D investment per firm,  $\tilde{l}^R$ , rather than, for instance, total R&D investment. This is an immediate implication of the assumption that the state of technology  $\tilde{S}_{t-1}$  is solely related to average product quality in  $t - 1$ . In contrast to per firm R&D outlay  $\tilde{l}^R$ , total R&D investment in equilibrium,  $\tilde{n}\tilde{l}^R$ , does not depend on the effectiveness of advertising,  $\eta$ , according to (16) and (18). As discussed in Section 4.1, the impact of an increase in  $\eta$  on  $\tilde{\vartheta}$  may change if  $\tilde{S}_{t-1}$  is not only determined by average product quality but also by the number of firms. As will become apparent, however, this introduces scale effects into the model.

<sup>6</sup>One should note that the positive relationship between firm size and R&D in the model also holds when firms are *ex ante* and *ex post* asymmetric, as discussed in the working paper version (Grossmann, 2003).

Third, one shall note that the positive size-R&D relationship is not in conflict with evidence on a potentially positive relationship between ‘competition and innovation’, as recently discussed by Aghion *et al.* (2005), among others. For instance, the Lerner index (price minus marginal cost divided by price) is simply given by  $(\tilde{p} - 1)/\tilde{p} = 1/\sigma$  in the model (recall  $\tilde{p} = \sigma/(\sigma - 1)$ ). Hence, according to this measure, an increase in product substitutability,  $\sigma$ , means more intensive product market competition. Interestingly, an increase in  $\sigma$  raises the elasticity of product demand with respect to R&D expenditure, thus implying higher R&D effort of firms,  $\tilde{l}^R$ . In sum, competition and innovation may be viewed as being positively linked, although firm size is positively related to R&D.

We now turn to the discussion of part (ii) of Proposition 2. Increases in  $\eta$  or  $\kappa$  affect welfare  $\tilde{U}$  in (20) through product number  $\tilde{n}$  and growth rate  $\tilde{v}$  in opposite directions (i.e.,  $\tilde{n}$  declines whereas  $\tilde{v}$  rises). The analysis suggests that the growth-enhancing impact of a higher effectiveness of marketing,  $\eta$ , may indeed dominate with respect to welfare effects (for instance, if  $\kappa$  is sufficiently high). Thus, although marketing activity is purely wasteful in the model (i.e., a social planner would choose  $l^M = 0$  at all times), it may well be positively related not only to growth but also to welfare in market equilibrium. One way to think about this is that the negative (static) externality from advertising mitigates the other market failures, arising from the positive (intertemporal) externality from R&D investments and imperfect product market competition. In the benchmark case without marketing ( $\eta = 0$ ), these imperfections imply that R&D investments per firm are inefficiently low and the number of firms is too high from a social planner’s perspective, as shown in Young (1998) and, for the specifications used here, Grossmann (2003). A higher  $\eta$  lowers the number of firms and raises R&D activity per firm, bringing both  $\tilde{n}$  and  $\tilde{l}^R$  closer to the social optimum.<sup>7</sup> Moreover, part (ii) of Proposition 2 reveals an additional interesting welfare implication of the interplay between advertising and R&D incentives. Whereas the impact of an increase in the effectiveness of R&D,  $\kappa$ , on welfare  $\tilde{U}$  is positive if the elasticity of product demand with respect to marketing effort,  $\eta(\sigma - 1)$ , is sufficiently low, a higher  $\kappa$  may be detrimental to welfare if  $\eta$  is high. Obviously, the latter result cannot arise in the benchmark case without advertising.

#### 4. Extensions

This section extends the model in two ways. First, it is examined how the introduction of scale effects with respect to growth alters the role of advertising incentives for growth and intertemporal welfare. Second, in view of the positive interaction between R&D and advertising effort of firms suggested by the preceding

<sup>7</sup> Also interestingly, an increase in  $\eta$  and an increase in fixed cost,  $f$ , qualitatively have the same effects. Again, this highlights the mechanism based on the relation between firm size ( $L/\tilde{n}$ ) and R&D investment per firm ( $\tilde{l}^R$ ) in the model, both of which are positively affected by  $f$  or  $\eta$ .

analysis, it is interesting to explore whether R&D subsidies, which are effective to promote R&D as will become apparent, affect advertising expenditure as well.

#### 4.1 Scale effects

Suppose now that in addition to the average product quality of firms in  $t - 1$ , also the number of firms affects the state of technology  $\bar{S}_{t-1}$ . Formally, the intertemporal knowledge spillover (5) is generalized to

$$\bar{S}_{t-1} = (n_{t-1})^{1-\varepsilon} \bar{q}_{t-1} = \frac{1}{(n_{t-1})^\varepsilon} \int_0^{n_{t-1}} q_{t-1}(i) di, \tag{21}$$

where  $0 \leq \varepsilon \leq 1$ . Consequently, according to (4) and (21), in symmetric equilibrium,  $\bar{S}_{t-1} = \bar{S}_{t-2} (n_{t-1})^{1-\varepsilon} (l_{t-2}^R)^\kappa$ . Thus, for  $\varepsilon < 1$ , the number of firms affects the economy's growth rate in equilibrium. We have seen that in the basic model, i.e., in special case  $\varepsilon = 1$ , the steady state growth rate does not depend on market size,  $L$ , i.e., there is no scale effect regarding growth.<sup>8</sup> But as the equilibrium number of firms,  $\tilde{n}$ , positively depends on population size  $L$ , according to (18), this means that there are scale effects regarding growth if  $\varepsilon < 1$ . Whereas the allocation of resources in market equilibrium remains unchanged, the following modifications regarding the effects of an increase in  $\eta$  arise.

*Proposition 3* (Scale effects) Under spillover effect (21), an increase in  $\eta$  raises (does not affect, lowers) the steady state growth rate if  $\kappa + \varepsilon > (=, <) 1$  and raises (does not affect, lowers) welfare if  $[\kappa - \rho(1 - \varepsilon)](\sigma - 1) + \rho > (=, <) 1$ .

According to Proposition 3, the stronger scale effects are (i.e., the lower  $\varepsilon$  is), the weaker is the potentially beneficial role of advertising incentives for raising growth or welfare. Indeed, in contrast to the case without scale effects ( $\varepsilon = 1$ ), advertising effort may be negatively related to growth if  $\varepsilon$  is low. Empirically, however, the support for scale effects is rather weak as discussed, e.g., in Jones (1995a,b) and Dinopoulos and Thompson (1999).

#### 4.2 R&D subsidies

This subsection deals with the question how R&D subsidies affect R&D and marketing expenditure of firms. Suppose each firm obtains a subsidy  $\tau$  per unit of R&D labor employed, where we follow the standard assumption that the subsidy is financed by lump-sum taxation of consumers. (Remark 2 in the Appendix shows that the government's budget is always balanced under this assumption.)

*Proposition 4* (R&D subsidies) An increase in R&D subsidy rate  $\tau$  raises both R&D investment per firm and growth, reduces the number of firms and does not affect advertising expenditure per firm.

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<sup>8</sup> However, as usual (see Jones, 1999), there is a positive scale effect with respect to the level of real consumption expenditure *per capita*,  $c_t = C_t/L$ .

An increase in the R&D subsidy rate  $\tau$  reduces marginal costs of R&D labor, and thus gives an incentive to firms to raise sunk cost for innovation activity. The net effect of an increase in  $\tau$  on the equilibrium number of firms is negative. Moreover, the amount of marketing labor per firm is not affected by a higher  $\tau$ , for the following reason. On the one hand, if  $\eta > 0$ , the two non-production activities within a firm are positively related, according to  $l_{t-1}^M/l_{t-1}^R = \eta(1 - \tau)/\kappa$ . As R&D investment increases with  $\tau$ , this implies a positive impact of an increase in  $\tau$  on marketing effort. On the other hand, however, an increase in  $\tau$  reduces marginal costs of R&D relative to those of advertising, which has a counteracting effect on advertising expenditure per firm. Both effects exactly cancel. Thus, R&D subsidies are conducive to growth without inducing further wasteful competition among firms through combative advertising.

In contrast to the positive impact of an increase of  $\tau$  on R&D investment and growth derived in Proposition 4, the analysis of Young (1998, p.52) suggests that, in absence of scale effects, ‘the provision of proportional R&D subsidies . . . will be ineffective (in growth rates)’. Clearly, this is not true in general. As shown in Grossmann (2003), the result in Young (1998) is driven by his assumption that there are no fixed costs for the production of goods,  $f=0$ , together with a different specification of the R&D technology.

## 5. Checking robustness in alternative frameworks

So far, the analysis has focussed on demand with a constant price elasticity as in Dixit and Stiglitz (1977). Although standard in R&D-based growth theory, the product demand structure is special in that output prices are independent of the number of firms. To check whether this special feature affects the main results, this section first introduces advertising and R&D into the linear-demand monopolistic competition framework of Ottaviano *et al.* (2002). Another special feature of the considered model concerns the absence of capital accumulation, as common in growth models with differentiated consumer products. Alternatively, we examine advertising on differentiated producer goods (intermediate inputs) in a framework where R&D investments interact with capital accumulation.

### 5.1 Linear demand

To extend the monopolistic competition framework with linear demand in Ottaviano *et al.* (2002) to a R&D-based growth model, suppose now that utility is given by  $U = \sum_{t=0}^{\infty} C_t/(1+r)^t$ , where the interest rate  $r > 0$  is exogenous (e.g., internationally given) and

$$C_t = \int_0^{n_t} q_t(i)x_t(i)di - \frac{\beta}{2} \int_0^{n_t} x_t(i)^2 di - \frac{\gamma}{2} \left( \int_0^{n_t} x_t(i)di \right)^2 + B_t Y_t, \quad (22)$$

$\beta, \gamma > 0$ .  $Y_t$  is the numeraire good supplied under perfect competition and  $B_t$  is an indicator of its quality at date  $t$ . Again, each firm  $i$  produces one variety of the differentiated good under monopolistic competition. For simplicity, marginal production cost of the differentiated good are set to zero, whereas one unit of the numeraire requires one unit of production labor. Thus, as there is perfect competition in the numeraire sector, for the wage rate we again have  $w_t = 1$ . Also for simplicity, let  $f=0$ . Quality  $q_t(i)$  and perceived quality  $q_t^*(i)$  of each differentiated good and the state of technology,  $\bar{S}_{t-1}$ , evolve like in section 2. As will become apparent, a steady state requires  $B_t = (\bar{S}_{t-1})^2$  for  $t \geq 1$ , which is assumed.<sup>9</sup> ( $B_0$  is given.)

According to (22), the inverse demand function for variety  $i$  is given by

$$p_t(i) = \frac{q_t^*(i) - \beta x_t(i) - \gamma X_t}{B_t}, \tag{23}$$

where  $X_t \equiv \int_0^{n_t} x_t(i) di$  is aggregate output of the differentiated good at date  $t \geq 0$ . Suppose that firms compete in quantities, taking  $X_t$  as given (following Ottaviano *et al.*, 2002). The first-order condition associated with maximizing

$$p_t(i)x_t(i) = \frac{q_t^*(i) - \beta x_t(i) - \gamma X_t}{B_t} x_t(i)$$

implies  $x_t(i) = 0.5(q_t^*(i) - \gamma X_t)/\beta$ . Integrating both sides over  $i \in [0, n_t]$  and solving for  $X_t$  yields  $X_t = Q_t(2\beta + \gamma n_t)^{-1}$ , where  $Q_t \equiv \int_0^{n_t} q_t^*(i) di$ . Substituting this back into the expression for  $x_t(i)$  we obtain

$$x_t(i) = \frac{1}{2\beta} \left( q_t^*(i) - \frac{\gamma Q_t}{2\beta + \gamma n_t} \right). \tag{24}$$

Using (23) one also obtains  $p_t(i) = \beta x_t(i)/B_t$ , implying equilibrium profits  $\pi_t(i) = \beta x_t(i)^2/B_t$ . Taking  $Q_t$  as given, firm  $i \in \mathcal{N}_t$  chooses  $l_{t-1}^R(i)$  and  $l_{t-1}^M(i)$  in  $t-1$  to maximize  $\pi_t(i)/(1+r) - l_{t-1}^R(i) - l_{t-1}^M(i)$ . Assuming strict concavity of the firms' objective functions, one can show that, again, the economy immediately jumps in a steady state and the following holds.

*Proposition 5* (Linear demand model) In symmetric steady state equilibrium, an increase in  $\eta$  raises both R&D and marketing expenditure per firm, but reduces the number of firms.

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<sup>9</sup> Such a relationship between the quality of the numeraire good and that of the differentiated good may either capture a spillover effect from the differentiated to the numeraire good sector or may reflect the notion that the rest of the economy, captured by the numeraire good sector, behaves similarly to the differentiated goods sector.

Proposition 5 shows that, qualitatively, the relationship between advertising incentives, firm size and R&D outlays is the same as in the previously analysed model which was characterized by a constant price elasticity of demand. An increase in the effectiveness of marketing,  $\eta$ , again raises firm size by inducing firms to incur higher sunk costs for advertising, in turn raising the return to R&D. As a consequence, the equilibrium growth rate increases when the state of technology evolves according to (5), as assumed in the basic model.

It can also be shown (available on request) that, despite a decrease in product variety, the net effect on utility may be positive like in the previous analysis, i.e., welfare may again be positively related to advertising.

## 5.2 Capital accumulation and intermediate goods

Now consider a monopolistic competition model in which there is a homogenous consumption good (the numeraire), produced with differentiated intermediate inputs of endogenous quality. Advertising affects demand of these producer goods. Output of the final good, produced under perfect competition, is given by<sup>10</sup>

$$Y_t = \left[ \int_0^{n_t} q_t(i)^{1-\alpha} x_t(i)^\alpha di \right] (L_t^P)^{1-\alpha}, \quad 0 < \alpha < 1, \quad (25)$$

where  $x_t(i)$  denotes the quantity of the intermediate input produced in firm  $i \in [0, n_t]$ ,  $q_t(i)$  is the productivity parameter associated with the latest version of that input and  $L_t^P$  is labor employed in final goods production. Analogously to the effect of advertising on perceived quality of consumer products, suppose that the final goods sector perceives quality  $q_t^*(i) = q_t(i)m_{t-1}(i)^\eta$  of producer durable  $i$ , where  $q_t(i)$  and  $\bar{S}_{t-1}$  again evolve like in (4) and (5), respectively.

Total capital stock  $K_t$  is defined as cumulative forgone output (following Romer, 1990). Abstracting from capital depreciation, we have

$$K_{t+1} = (1 + r_t)K_t + w_tL - C_t, \quad (26)$$

where  $r_0$  and  $K_0 > 0$  are given. Under intertemporal utility  $U = \sum_{t=0}^{\infty} \rho^t \ln C_t$  as in the basic model (there, however,  $C_t$  was a consumption index of differentiated products rather than the level of a homogenous consumption good), we obtain Euler equation

$$C_t = (1 + r_t)\rho C_{t-1}. \quad (27)$$

Following Romer (1990), suppose intermediate goods producers can transform  $\chi$  units of foregone consumption into one unit of any type of producer durable.

<sup>10</sup>This functional form follows Aghion, Howitt, and Mayer-Foulkes (2005). See also Romer (1990) for a similar type of production function.

Thus, marginal production costs are equal to  $\chi r_t$  and the measure of capital stock reads  $K_t = \chi \int_0^{\eta_t} x_t(i) di$ . According to (25), quality improvements lead to output expansion and therefore to capital accumulation. In other words, capital accumulation is fueled by R&D investments.

Using (25), wage rate  $w_t = (1 - \alpha)Y_t/L_t^P$  and the inverse demand function for intermediate input  $i$  is given by

$$p_t(i) = \alpha \left( \frac{q_t^*(i)L_t^P}{x_t(i)} \right)^{1-\alpha}. \tag{28}$$

Using (28), it is easy to show that maximization of profits  $\pi_t(i) = (p_t(i) - \chi r_t)x_t(i)$  leads to prices  $p_t(i) = \chi r_t/\alpha$ , which imply  $x_t(i) = (\alpha^2/\chi r_t)^{\frac{1}{1-\alpha}} q_t^*(i)L_t^P$ . Hence, using (4) and  $\pi_t(i) = (1/\alpha - 1)\chi r_t x_t(i)$ , firm  $i$  solves in  $t - 1$ :

$$\max_{l_{t-1}^R(i), l_{t-1}^M(i) \geq 0} \frac{(1 - \alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}}{(1 + r_t)(\chi r_t)^{\frac{\alpha}{1-\alpha}}} L_t^P \bar{S}_{t-1} l_{t-1}^R(i)^\kappa \left( \frac{l_{t-1}^M(i)}{\bar{l}_{t-1}^M} \right)^\eta - w_{t-1}(l_{t-1}^R(i) + l_{t-1}^M(i) + f). \tag{29}$$

The objective function is strictly concave if  $\kappa + \eta < 1$ . As shown in the proof of the next result, in this case there again exists a symmetric steady state equilibrium without transitional dynamics, in which  $r, l^R, l^M, L^P$  and  $n$  are time-invariant.

*Proposition 6* (Capital and intermediate goods) The result in Proposition 5 again holds. Moreover, for  $t \geq 0$ ,

$$\frac{\bar{S}_{t+1}}{\bar{S}_t} = \frac{K_{t+1}}{K_t} = \frac{Y_{t+1}}{Y_t} = \frac{C_{t+1}}{C_t} = \frac{w_{t+1}}{w_t} = \left( \frac{\kappa f}{1 - \kappa - \eta} \right)^\kappa. \tag{30}$$

Hence, the growth rate still does not exhibit scale effects and increases in firms' incentives to advertise producer goods,  $\eta$ . Invoking the insights developed in Section 4.1, one can again conclude that both of these properties of the model are linked. Thus, there remains the caveat that advertising may not be positively related to growth if the nature of intertemporal spillovers implies scale effects.

## 6. Conclusion

In a world with complex and differentiated goods, which are subject to ongoing quality improvements within firms, the physical attributes of a product are often difficult to ascertain for consumers before purchasing it. This leaves room for uninformative and socially wasteful marketing activities, redistributing demand from low- to high-advertisers.

This paper has analysed the implications of such combative advertising in a quality-ladder model of endogenous growth, which is characterized by two empirically supported features: a positive interaction between marketing expenditure and firm size on the one hand and a positive relationship between firm size and R&D due to cost spreading size advantages on the other hand. The analysis suggests that,



due to these features, R&D investments per firm are unambiguously positively related to incentives of firms to engage in combative advertising. Consequently, the economy's growth rate is increasing in the effectiveness of marketing except in the case that growth is subject to strong scale effects. Regarding growth policy, R&D subsidies are always conducive for growth in the model, without giving further incentives for firms to raise their advertising expenditure.

From a normative point of view, the analysis suggests the — at the first glance somewhat paradoxical — result that even purely combative advertising may be welfare-enhancing. This possibility arises because advertising can mitigate market failures associated with positive knowledge spillovers and imperfect goods markets.

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## Appendix

*Proof of Lemma 1* According to (6) and (7), aggregate product demand is given by  $n_t x_t^D = E_t / \tilde{p}$  in symmetric equilibrium. Using Euler equation (9) and expression  $\tilde{p} = \sigma / (\sigma - 1)$  then leads to

$$\frac{\tilde{p} - 1}{1 + r_t} x_t^D = \frac{\rho E_{t-1}}{\sigma n_t}. \quad (\text{A.1})$$

Thus, using  $(\tilde{p} - 1)x_t^D / (1 + r_t) = l_{t-1}^R + l_{t-1}^M + f$  from free entry condition (E3) yields

$$n_t (l_{t-1}^R + l_{t-1}^M + f) = \frac{\rho}{\sigma} E_{t-1}, \quad (\text{A.2})$$

$t \geq 1$ . Moreover, equating aggregate output,  $n_{t-1}l_{t-1}^p$ , with aggregate product demand,  $E_{t-1}/\tilde{p}$ , and using  $\tilde{p} = \sigma/(\sigma - 1)$  implies

$$n_{t-1}l_{t-1}^p = E_{t-1} \frac{\sigma - 1}{\sigma}. \tag{A.3}$$

Using (A.2) and (A.3), the labor market clearing condition (E5) implies that aggregate consumption spending is given by

$$E_{t-1} \equiv \tilde{E} = \frac{\sigma L}{\sigma - 1 + \rho}, \tag{A.4}$$

for all  $t \geq 1$ . (9) then confirms that the interest rate factor is given by  $1 + r_t = 1/\rho$  for all  $t \geq 1$ .  $\square$

*Proof of Proposition 1* First, substitute (A.1) into (12) which yields

$$\frac{\rho \tilde{E}_{t-1}}{\sigma n_t} \frac{\kappa}{l_{t-1}^R} = 1. \tag{A.5}$$

Substituting (A.2) into (A.5) and, for  $\eta > 0$ , also (14), and rearranging terms proves (16). (If  $\eta = 0$ , set  $l_{t-1}^M = 0$  in (A.2).) To find (17), use (14) and (16). Moreover, substituting (16) and (A.4) into (A.5), and rearranging terms gives (18). This confirms part (i). To prove part (ii), first note that symmetry implies that  $C_t = (n_t)^{\frac{\sigma}{\sigma-1}} \bar{S}_{t-1} (l_{t-1}^R)^\kappa l_t^p$ , according to (2)–(4). Thus, using the fact that, for  $t \geq 0$ ,  $n_t l_t^p = E_t (\sigma - 1)/\sigma$  from (A.3) and  $E_t = \tilde{E}$ , we obtain

$$C_t = \frac{\sigma - 1}{\sigma} \bar{S}_{t-1} (n_t)^{\frac{1}{\sigma-1}} (l_{t-1}^R)^\kappa \tilde{E}, \tag{A.6}$$

$t \geq 1$ . Also note that, under symmetry,  $\bar{S}_{t-1} = \bar{S}_{t-2} (l_{t-2}^R)^\kappa$ , according to (5), and thus,

$$\bar{S}_{t-1} = \bar{S}_0 (l_{t-2}^R)^\kappa (l_{t-3}^R)^\kappa \times \dots \times (l_1^R)^\kappa (l_0^R)^\kappa, \tag{A.7}$$

$t \geq 2$ . Substituting (A.7) into (A.6), and using  $l_{t-1}^R = \tilde{l}^R$  and  $n_t = \tilde{n}$  for all  $t \geq 1$  implies

$$C_t = \frac{\sigma - 1}{\sigma} \bar{S}_0 \tilde{n}^{\frac{1}{\sigma-1}} (\tilde{l}^R)^{\kappa t} \tilde{E} \tag{A.8}$$

for any  $t \geq 1$ . The first equation in (19) follows from (A.8); and substituting (16) into it also confirms the second one. To prove part (iii), first, derive analogously to (A.8) that

$$C_0 = \frac{\sigma - 1}{\sigma} \bar{S}_0 (n_0)^{\frac{1}{\sigma-1}} \tilde{E}, \tag{A.9}$$

by observing  $q_0(i) = \bar{S}_0$  for all  $i \in \mathcal{N}_0$ . Substituting (A.8) and (A.9) into (1) leads to

$$\begin{aligned} \tilde{U} &= \frac{\ln n_0}{\sigma - 1} + \sum_{t=0}^{\infty} \rho^t \ln \left( \frac{(\sigma - 1) \bar{S}_0 \tilde{E}}{\sigma} \right) + \sum_{t=1}^{\infty} \rho^t \left( \frac{\ln \tilde{n}}{\sigma - 1} + \kappa t \ln \tilde{l}^R \right) \\ &= \frac{\ln n_0}{\sigma - 1} + \frac{1}{1 - \rho} \left( \ln \left( \frac{(\sigma - 1) \bar{S}_0 \tilde{E}}{\sigma} \right) + \frac{\rho \ln \tilde{n}}{\sigma - 1} + \frac{\rho}{1 - \rho} \kappa \ln \tilde{l}^R \right), \end{aligned}$$

where  $\sum_{t=0}^{\infty} \rho^t = (1 - \rho)^{-1}$  and  $\sum_{t=1}^{\infty} \rho^t t = \rho(1 - \rho)^{-2}$  have been used for the latter equation. Substituting (A.4) into (A.10) and observing (19) gives (20). This concludes the proof.  $\square$

*Proof of Proposition 2* Part (i) immediately follows from (16)-(19). To prove part (ii), first, substitute (18) and (19) into (20). From this, it is easy to show that

$$\frac{\partial \tilde{U}}{\partial \kappa} = \frac{\rho}{(1-\rho)^2} \left( \frac{\rho - \eta(\sigma - 1)}{1 - (\kappa + \eta)(\sigma - 1)} + \ln \tilde{l}^R \right), \tag{A.11}$$

where  $\tilde{l}^R$  is given by (16), and

$$\frac{\partial \tilde{U}}{\partial \eta} = - \frac{\rho}{(1-\rho)^2} \frac{1 - \kappa(\sigma - 1) - \rho}{1 - (\kappa + \eta)(\sigma - 1)}, \tag{A.12}$$

respectively. Observing assumption (11) and noting that  $\tilde{l}^R > 1$  if  $f > \hat{f}$ , according to (19), then also confirms part (ii).  $\square$

*Remark 1* As argued in the discussion of part (i) of Proposition 2, if the number of firms were exogenous, R&D expenditure  $l^R$  would be a decreasing rather than an increasing function of  $\eta$ . To see this, first substitute  $E_{t-1}/\sigma = n_{t-1}l_{t-1}^p/(\sigma - 1)$  from equation (A.3) into (A.1), and use  $n_{t-1} = n_t = n$ , to obtain  $(\tilde{p} - 1)(\sigma - 1)x_t^D/(1 + r_t) = \rho l_{t-1}^p$ . Substituting this expression into (12) leads to  $l_{t-1}^R = \rho \kappa l_{t-1}^p$ ,  $t \geq 1$ . Finally, substituting both  $l_{t-1}^p = l_{t-1}^R/(\rho \kappa)$  and  $l_{t-1}^M = l_{t-1}^R \eta/\kappa$  into the labor market clearing condition  $n(l_{t-1}^p + l_{t-1}^R + l_{t-1}^M + f) = L$  yields  $l_{t-1}^R = \frac{\rho \kappa(L/n-f)}{1 + \rho(\kappa + \eta)}$  for all  $t \geq 1$ . Thus,  $l_{t-1}^R$  is decreasing in  $\eta$ .  $\square$

*Proof of Proposition 3* In equilibrium, for all  $t \geq 1$ ,  $l_{t-1}^R(i) = \tilde{l}^R$  for all  $i \in \mathcal{N}_t$ , and  $n_t = \tilde{n}$ . Thus, (21) implies

$$\tilde{S}_{t-1} = \tilde{S}_{t-2}(l_{t-2}^R)^\kappa (n_{t-1})^{1-\varepsilon} = \tilde{S}_0(\tilde{l}^R)^{\kappa(t-1)} \tilde{n}^{(t-1)(1-\varepsilon)} \tag{A.13}$$

for any  $t \geq 2$ . Substituting (A.13) into (A.6) implies, for any  $t \geq 1$ ,

$$C_t = \frac{\sigma - 1}{\sigma} \tilde{S}_0(\tilde{l}^R)^{\kappa t} \tilde{n}^{\frac{1}{\sigma-1}} \tilde{n}^{(t-1)(1-\varepsilon)} \tilde{E} \tag{A.14}$$

(whereas  $C_0$  is still given by (A.9)). Thus, the steady state growth rate is given by  $\tilde{\vartheta} = \kappa \ln \tilde{l}^R + (1 - \varepsilon) \ln \tilde{n}$ . Moreover, by substituting (A.9) and (A.14) into (21) and observing  $\sum_{t=1}^\infty \rho^t (t - 1) = \rho^2(1 - \rho)^{-2}$ , it is easy to show that intertemporal welfare is now given by  $\tilde{U} + \rho^2(1 - \rho)^{-2}(1 - \varepsilon) \ln \tilde{n}$ , where  $\tilde{U}$  is given by (A.10). From these expressions, and by using (16) and (18), Proposition 3 is easily confirmed.  $\square$

*Proof of Proposition 4* With an R&D subsidy rate  $\tau$  and  $p_t(i) = \tilde{p}$ , the firm value becomes

$$\frac{\tilde{p} - 1}{1 + r_t} x_t^D(i) - (1 - \tau)l_{t-1}^R(i) - l_{t-1}^M(i) - f. \tag{A.15}$$

Using  $x_t^D(i) = x_t^D$  and (A.1) one finds analogously to the first-order conditions (12), (13), and the free entry condition (E3), that

$$\frac{\rho E_{t-1}}{\sigma n_t} (\sigma - 1) \frac{\kappa}{l_{t-1}^R} = 1 - \tau, \tag{A.16}$$

$$\frac{\rho E_{t-1}}{\sigma n_t} (\sigma - 1) \frac{\eta}{l_{t-1}^M} = 1 \tag{A.17}$$

and

$$\frac{\rho E_{t-1}}{\sigma n_t} = (1 - \tau)l_{t-1}^R + l_{t-1}^M + f, \tag{A.18}$$

respectively. Moreover, using (A.3), the labor market clearing condition (E5) implies

$$\frac{\sigma - 1}{\sigma} E_{t-1} + n_t (l_{t-1}^R + l_{t-1}^M + f) = L. \tag{A.19}$$

Using (A.16)-(A.19) one obtains after some manipulations:

$$n_t = \frac{\sigma L}{f} \frac{[1 - (\kappa + \eta)(\sigma - 1)](1 - \tau)}{(\sigma - 1)(1 - \tau) + \rho[1 - \tau(1 - \kappa(\sigma - 1))]}, \tag{A.20}$$

$$l_{t-1}^R = \frac{\kappa(\sigma - 1)f}{(1 - \tau)[1 - (\kappa + \eta)(\sigma - 1)]}, \tag{A.21}$$

$$l_{t-1}^M = \frac{\eta(\sigma - 1)f}{1 - (\kappa + \eta)(\sigma - 1)} \tag{A.22}$$

for all  $t \geq 1$ . Observing assumption (11), Proposition 4 immediately follows from (A.20)-(A.22).  $\square$

*Remark 2* It remains to confirm that the government’s budget is always balanced if the (possibly negative) R&D subsidy is financed by a lump-sum tax (or subsidy) of consumers, denoted by  $T_t$  at date  $t$ . To see this, first note that budget constraint (8) modifies to

$$A_{t+1} = (1 + r_t)A_t + L - E_t - T_t, \tag{A.23}$$

$t \geq 0$ , where  $A_t$  denotes the value of assets in  $t$ . Due to the borrowing of firms, at any date  $t \geq 0$  we have

$$A_{t+1} = n_{t+1} [(1 - \tau)l_t^R + l_t^M + f]. \tag{A.24}$$

Thus, under free entry, which implies that the expression in (A.15) equals zero in equilibrium, making use of (A.1) gives us  $A_t = \rho E_t / \sigma$ . Using  $1 + r_t = 1/\rho$  from Lemma 1 (which also holds with R&D subsidies), this leads to  $(1 + r_t)A_t = E_t / \sigma$ . Also note that labor market clearing at date  $t \geq 0$  requires  $n_t l_t^P + n_{t+1} (l_t^R + l_t^M + f) = L$ . Substituting the latter two expressions into (A.23),

$$A_{t+1} = \frac{E_t}{\sigma} + n_t l_t^P + n_{t+1} (l_t^R + l_t^M + f) - E_t - T_t. \tag{A.25}$$

Finally, substituting (A.24) and  $n_t l_t^P = (\sigma - 1)E_t / \sigma$  from (A.3) into (A.25), one obtains  $T_t = \tau n_{t+1} l_t^R$ , i.e., the lump-sum tax equals the required total government spending at any date  $t \geq 0$ . This proves the claim.  $\square$

*Proof of Proposition 5* In  $t - 1$ , firm  $i \in \mathcal{N}_t$  solves

$$\max_{l_{t-1}^R(i), l_{t-1}^M(i)} \left\{ \frac{\left[ \bar{S}_{t-1} l_{t-1}^R(i) \left( \frac{l_{t-1}^M(i)}{l_{t-1}^R(i)} \right)^\eta - \frac{\gamma Q_t}{2\beta + \gamma n_t} \right]^2}{4\beta(1+r)B_t} - l_{t-1}^R(i) - l_{t-1}^M(i) \right\}, \tag{A.26}$$

according to  $\pi_t(i) = \beta x_t(i)^2/B_t$  and (24), by observing (4). Under symmetry (where  $q_t^*(i) = q_t = \bar{S}_{t-1}(l_{t-1}^R)^\kappa$  for all  $i$ ), the first-order conditions corresponding to (A.26) imply

$$\kappa q_t x_t = (1+r)l_{t-1}^R B_t, \tag{A.27}$$

$$l_{t-1}^M = \frac{l_{t-1}^R \eta}{\kappa}. \tag{A.28}$$

Moreover, under symmetry,  $Q_t = n_t q_t$  and thus

$$x_t = \frac{q_t}{2\beta + \gamma n_t}, \tag{A.29}$$

according to (24). Substituting (A.29) into (A.27) and using (4) yields

$$\kappa(\bar{S}_{t-1})^2 = (1+r)(l_{t-1}^R)^{1-2\kappa}(2\beta + \gamma n_t)B_t. \tag{A.30}$$

Moreover, note that  $\beta(x_t)^2/B_t = (1+r)(l_{t-1}^R + l_{t-1}^M)$  under free entry. Substituting (A.28) and (A.29) into this expression and again using (4) implies

$$\beta\kappa(\bar{S}_{t-1})^2 = (1+r)(l_{t-1}^R)^{1-2\kappa}(2\beta + \gamma n_t)^2(\kappa + \eta)B_t. \tag{A.31}$$

Combining (A.30) and (A.31), we obtain

$$n_t = \frac{\beta(1 - 2(\kappa + \eta))}{\gamma(\kappa + \eta)}, \tag{A.32}$$

for all  $t \geq 1$  (i.e., existence of a symmetric equilibrium requires  $\kappa + \eta < 1/2$ ). Thus, using steady state condition  $B_t = (\bar{S}_{t-1})^2$ , we find

$$l_{t-1}^R = \left( \frac{\kappa(\kappa + \eta)}{\beta(1+r)} \right)^{\frac{1}{1-2\kappa}}, \tag{A.33}$$

$t \geq 1$ . Using (A.28), (A.32) and (A.33) then confirms Proposition 5. □

*Proof of Proposition 6* First-order conditions associated with (29) imply, under symmetry,

$$\frac{(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}}{(1+r_t)(\chi r_t)^{\frac{\alpha}{1-\alpha}}} L_t^P \kappa (l_{t-1}^R)^{\kappa-1} = \omega_{t-1}, \tag{A.34}$$

where  $\omega_{t-1} \equiv w_{t-1}/\bar{S}_{t-1}$ , and  $l_{t-1}^M = (\eta/\kappa)l_{t-1}^R$ . Moreover, since firm value as given by (29) must be zero under free entry,

$$\frac{(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}}{(1+r_t)(\chi r_t)^{\frac{\alpha}{1-\alpha}}} L_t^P (l_{t-1}^R)^\kappa = \omega_{t-1}(l_{t-1}^R + l_{t-1}^M + f) \tag{A.35}$$

Combining (A.34) and (A.35) and using  $l_{t-1}^M = (\eta/\kappa)l_{t-1}^R$  leads to

$$l_{t-1}^R = \frac{\kappa f}{1 - \kappa - \eta}, \tag{A.36}$$

$t \geq 1$ . This confirms the results regarding  $l_{t-1}^R$  and  $l_{t-1}^M$ , which are time-invariant. One can therefore look for a steady state equilibrium in which also  $L^P$ ,  $r$ ,  $n$ , and therefore  $\omega$  are time-invariant. Dropping time index  $t$  whenever possible, substituting  $p(i) = \chi r/\alpha$  into (28) and observing  $q_t^*(i) = q_t(i)$  implies

$$x_t(i) = \left(\frac{\alpha^2}{\chi r}\right)^{\frac{1}{1-\alpha}} q_t(i)L^P \tag{A.37}$$

and thus, according to (25),

$$Y_t = \left(\frac{\alpha^2}{\chi r}\right)^{\frac{1}{1-\alpha}} L^P \int_0^{n_t} q_t(i)di. \tag{A.38}$$

Combining (A.38) with  $w_{t-1} = (1 - \alpha)Y_{t-1}/L^P$  and using  $n_t \bar{S}_{t-1} = \int_0^{n_t} q_t(i)di$  from (5) one obtains

$$\omega_{t-1} = \omega = (1 - \alpha) \left(\frac{\alpha^2}{\chi r}\right)^{\frac{1}{1-\alpha}} nL^P. \tag{A.39}$$

Combining (A.34) and (A.39) implies  $L^P = n(l^R)^{1-\kappa}(1+r)/(\alpha\kappa)$ . Substituting this expression together with  $l^M = (\eta/\kappa)l^R$  into labor market clearing condition  $L^P + n(l^R + l^M + f) = L$  leads to

$$n = \frac{L}{\frac{f}{1-\kappa-\eta} + \frac{(l^R)^{1-\kappa}(1+r)}{\alpha\kappa}}. \tag{A.40}$$

Given that  $l^R$  as given in (A.36) increases with  $\eta$ , the number of firms  $n$  decreases with  $\eta$  if  $r$  is non-decreasing in  $\eta$ . This as well as (30) is confirmed next. Since  $\bar{S}_{t+1}/\bar{S}_t = (l^R)^\kappa$ , according to (5),  $q_t(i) = \bar{S}_{t-1}(l^R)^\kappa$  implies that also  $x_t(i)$ ,  $K_t$ ,  $Y_t$  and  $w_t = \bar{S}_{t-1}\omega$  grow at the same rate. With capital accumulating according to (26), this implies that  $C_{t+1}/C_t = (l^R)^\kappa$ . Using (A.36), this confirms (30). Moreover, using Euler equation (27), we also have  $C_{t+1}/C_t = \rho(1+r)$ . Hence,  $\rho(1+r) = (l^R)^\kappa$ , which gives us  $r$  as increasing function of  $\eta$ . This confirms that, in equilibrium,  $n$  is a decreasing function of  $\eta$ . This concludes the proof.  $\square$